Scriptless Bitcoin Lotteries from Oblivious Transfer

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What’s a Lottery?

Imagine a trusted party Lloyd who conducts lotteries between Alice and Bob:

- Alice and Bob send 1 BTC to Lloyd’s Address
- Lloyd chooses a winner randomly and sends them 2 BTC

The goal of a lottery protocol is achieve the same result without a trusted third party.
What’s Scriptless?

Bitcoin has a way of specifying the spending rules on coins with a language called “Script”. A scriptless protocol doesn’t use it. Instead it realises the spending rules off-chain through some cryptographic protocol. For example:

- You can replace OP_CHECKMULTISIG with a threshold multi-signature scheme.
- You can replace HTLC by using “adaptor” signatures.
Why Scriptless?

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To know what can and can’t be done without script

https://bitcointalk.org/index.php?topic=355174.0

...I think the multi-party lottery scheme still rates as the most advanced usage of script yet found in the wild

— Mike Hearn
How to do a lottery without a trusted third party?

- Need to fairly generate randomness (no OP_RANDOM)
- The randomness choose the winner
- Enforce the outcome on the blockchain

All previous lotteries used hash commitment “coin tossing” to generate the randomness
Idea introduced in by Manuel Blum in 1981: *Coin Flipping By Telephone: A protocol for solving impossible problems*

*Alice and Bob want to flip a coin by telephone. (They have just divorced, live in different cities, want to decide who gets the car.)*
Coin tossing

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*Alice and Bob want to flip a coin by telephone. (They have just divorced, live in different cities, want to decide who gets the car.)*

**basic idea:**

1. Alice sends a “commitment” to her coin toss
2. Bob sends his coin toss
3. Alice reveals her coin toss
Hash-Commitment Coin tossing

\[\begin{align*}
\text{Alice} & \quad \text{Bob} \\
 b_1 & \leftarrow \{0, 1\} \\
r & \leftarrow \{0, 1\}^l \\
t & \leftarrow H(b_1, r) \\
& \quad t \\
& \quad \quad b_2 \\
& \quad \quad \quad \leftarrow \{0, 1\} \\
& \quad \quad \quad \leftarrow b_1 \oplus b_2 \\
& \quad \quad \quad \quad b'_1, r' \\
& \quad \quad \quad \quad \quad \quad H(b'_1, r') \overset{?}{=} t \\
& \quad \quad \quad \quad \quad \quad b'_1 \oplus b_2 \Rightarrow
\end{align*}\]

Note: It's \textit{unfair}

Lloyd Fournier (Scaling Bitcoin 2019)
Lottery Protocols – First Attempt

Iddo - fair coin toss with no extortion and no need to trust a third party

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system that doesn’t involve a 3rd-party like SatoshiDice, so we
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Two Ideas:

- Iddo Bentov: We can use collateral to force both parties to reveal pre-images.
- Adam Back: If a party doesn’t reveal their pre-image we should just make them lose by default.
Zero-Collateral Lotteries in Bitcoin and Ethereum

Tx: Alice and Bob bail in
- Left Input
- Right Input

Require N-N multisig
If T1 < Block:
  ok
If Block < T1:
  Reveal a, such that A = H(a)

Tx: Alice reveals the value a
- Reveal a where A = H(a)

Require N-N multisig
If T1 < Block < T2:
  Reveal b, such that B = H(b)
  Reveal a, such that A = H(a)
  Assert (a^b) == 1 mod 2
If T2 < Block < T3:
  ok

Outcome 1: Bob Wins (Alice aborts)
Outcome 2: Bob Wins
Reveal a, b, s.t.
A = H(a) and B = H(b) and
a^b == 1 mod 2
How do we get our lottery to look like this?
Oblivious transfer

Manuel Blum to the rescue (again)! He made another discovery in 1981: you can generate a random outcome with oblivious transfer.


Rough Definition: Alice transmits one of $n$ messages to Bob of Bob's choosing (But Alice doesn't learn which message).
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Rough Definition:
- Receiver security: Sender doesn't know which message was sent.
- Sender security: Receiver only gets one message.
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Rough Definition:

1. Alice transmits one of $n$ message to Bob of Bob’s choosing (But Alice doesn’t learn which message).

2. **receiver security** Sender doesn’t know which message was sent

3. **sender security** Receiver only gets one message
\[ m_0, m_1 \leftarrow \$ \{0, 1\}^k \]

\[ b_2 \leftarrow \$ \{0, 1\} \]

\[ \leftrightarrow \text{OT}((b_2), (m_0, m_1)) \leftrightarrow \]

\[ \text{learns } m_{b_2} \]

\[ b_1 \leftarrow \$ \{0, 1\} \]

\[ \begin{align*}
    m' &= m_0 \\
    m' &= m_1 \\
    \perp &= \text{otherwise}
\end{align*} \]

\[ b'_2 := \begin{cases}
0 & m' = m_0 \\
1 & m' = m_1 \\
\perp & \text{otherwise}
\end{cases} \]

\[ \leftarrow b_1 \oplus b'_2 \]
Choose keys securely

Sign the transaction scaffold

Alice obliviously signs BobWin

Sign and broadcast Fund

Scriptless Lotteries

September 10, 2019 13 / 23
1. Choose keys securely

2. Sign the transaction scaffold

3. Alice obliviously signs BobWin

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Scriptless Lotteries
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1. Choose keys securely
2. Sign the transaction scaffold
3. Alice *obliviously signs* BobWin
4. Sign and broadcast Fund
How to realise Oblivious Signing?

We want Bob to have a signature on BobWin_0 OR BobWin_1 but without Alice knowing which one.
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**Approach**: Use adaptor signatures where Bob only knows the completion for one of them.
Adaptor signatures enable the signer to create a simple access structure to a particular signature on a particular message.

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<th>Adaptor</th>
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<tbody>
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<td>args</td>
<td>$sk, m$</td>
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**Approach:** Alice to give Bob adaptor signatures where Bob only knows the completion for one of them.

A Pedersen commitment for $x$ is in the form $T = g^x h^c$. It's impossible to decommit to more than one value without knowing the DLOG $g(h)^c$. Fix $c \in \{0, 1\}$, the committer can only know the discrete log of $T$ if $c = 0 \rightarrow T = g^x$ (knows DLOG ($T$)) or $c = 1 \rightarrow T = g^x h$ (knows DLOG $(Th - 1)^c$).
Oblivious Signing

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- It’s impossible to decommit to more than one value without knowing the DLOG$_g(h)$
- Fix $c \in \{0, 1\}$, committer can only know discrete log of $T$ OR $Th^{-1}$
  - $c = 0 \rightarrow T = g^x$ (knows DLOG$(T)$)
  - $c = 1 \rightarrow T = g^x h$ (knows DLOG$(Th^{-1})$)
Oblivious Signing

\[\begin{align*}
\text{Alice}(sk) & \quad (m_0, m_1, h) & \text{Bob}(pk, c \in \{0, 1\}) \\
 y \leftarrow & \mathbb{Z}_q \\
 T & = g^y h^c \\
 (s_0, R_0) & = \sigma(sk, m_0, T) \\
 (s_1, R_1) & = \sigma(sk, m_1, Th^{-1}) \\
 (s_0, R_0), (s_1, R_1) & \leftarrow (s_c + y, R_c \cdot g^y) \Rightarrow
\end{align*}\]
Secure?

1 security for sender (Alice): If Bob can complete both adaptor signatures then he can solve $DLOG(h)$ (from the completion of both adaptor signatures we learn $DLOG$ of $T, Th^{-1}$).
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Not so fast! Our transactions outputs are locked by joint public keys!
Two-party oblivious signing

One party knows which of two messages they are both signing but the other doesn’t. Make a few changes:

1. They first choose joint nonces $R_{\text{BobWin}_0}, R_{\text{BobWin}_1}$ (we did this already).
2. Along with $T$ Bob sends half adaptor signatures for $m_0$ ($\text{BobWin}_0$) and $m_1$ ($\text{BobWin}_1$) under $B_0$ and $B_1$
3. Alice calculates the full adaptor signatures and sends them back to Bob
Lotteries were previously an example of something that could only be done with non-trivial script.

Oblivious signing with Schnorr signatures is secure and efficient (at least as it is used here).

Unsubstantiated claims:
- You can do lotteries with different odds by doing $1/N$ oblivious signing rather than $1/2$.
- You can cooperatively complete the lottery in two on-chain transactions.
- You can execute cooperative protocol in a payment channel (and even do it "multi-hop" I think).

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