

# Threshold Scriptless Scripts

Omer Shlomovits



# Scriptless Scripts

"Magicking digital signatures so that they can only be created by faithful execution of a smart contract".

Andrew Poelstra

*Scriptless Scripts*

Andrew Poelstra

`grindelwald@wpsoftware.net`

May 10, 2017

# In This Talk...

- Intro to Schnorr Scriptless Scripts (SSS)
- The road to ECDSA Scriptless Scripts (ESS)
  - Rebuttal ECDSA known security issues
  - Discussing 2P-ECDSA as main tool
  - Expanding the Tool Box with Threshold ECDSA
- Experimenting with ESS

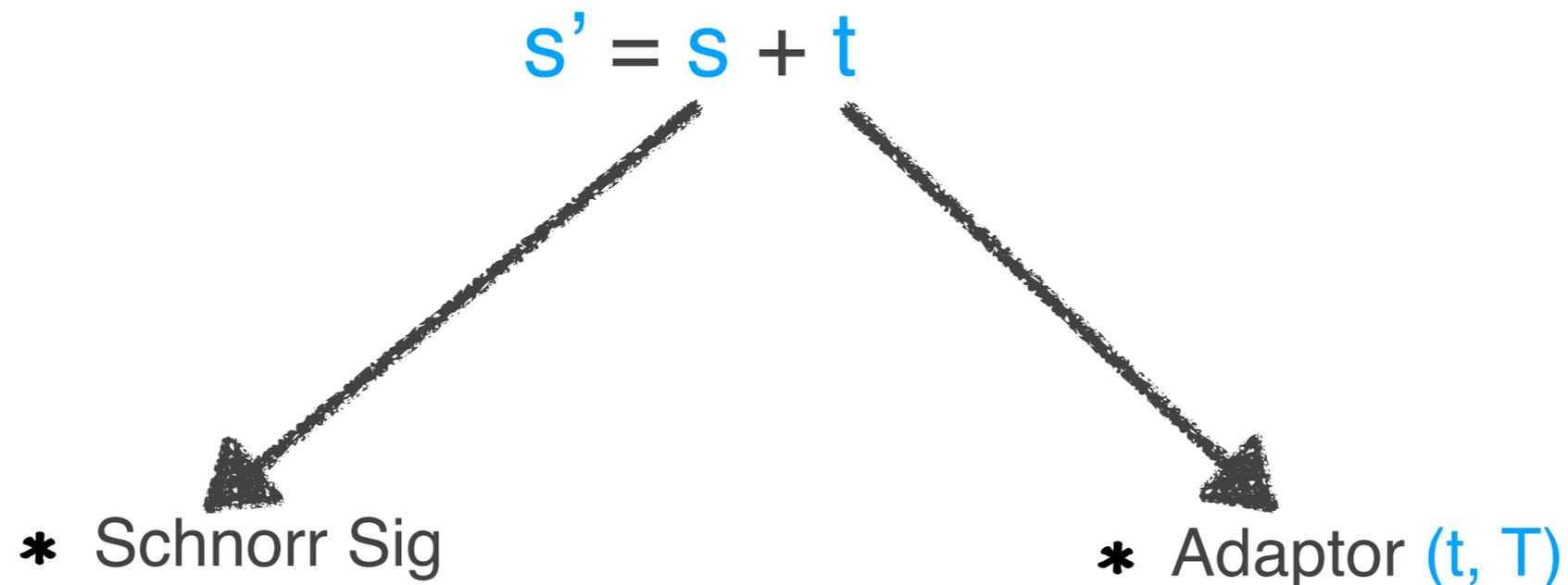
# Schnorr Signature

EC public parameters :  $q, G$

- Choose random  $k$
- Compute  $R = k \cdot G$
- Compute  $s = k + H(R, P, m) \cdot x \pmod q$   
where  $x$  is the private key,  $R = x \cdot G$
- Output  $(R, s)$

# The Shtik

- \* Adaptor Signature: Main building block is a tweak to Schnorr signature ( $R, s$ )



shtik 



(Yiddish) a devious trick;



[ElementsProject/scriptless-scripts](https://github.com/ElementsProject/scriptless-scripts)

# SS Atomic Swap



Wallet A

$PK_A + PK_B$

Wallet B

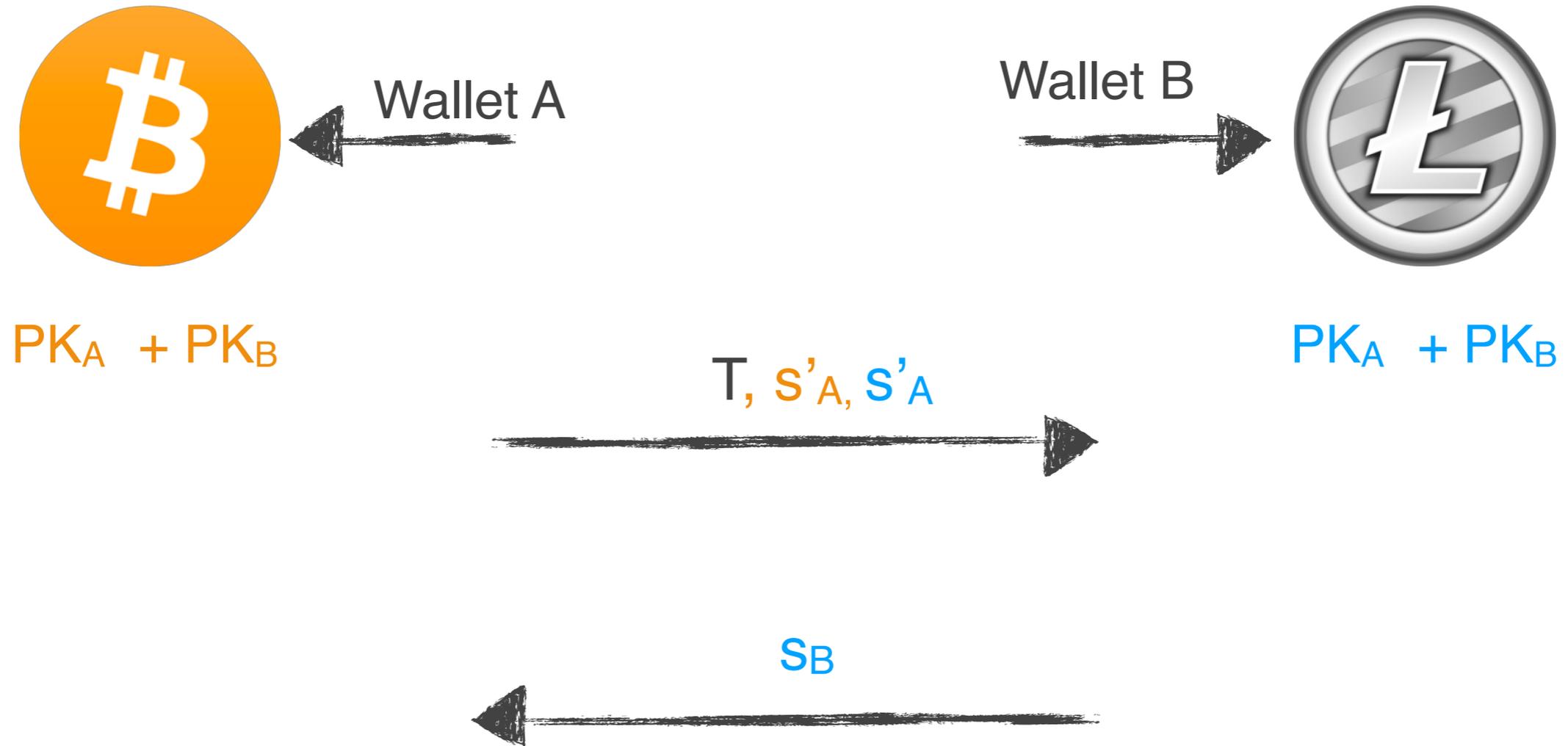


$PK_A + PK_B$

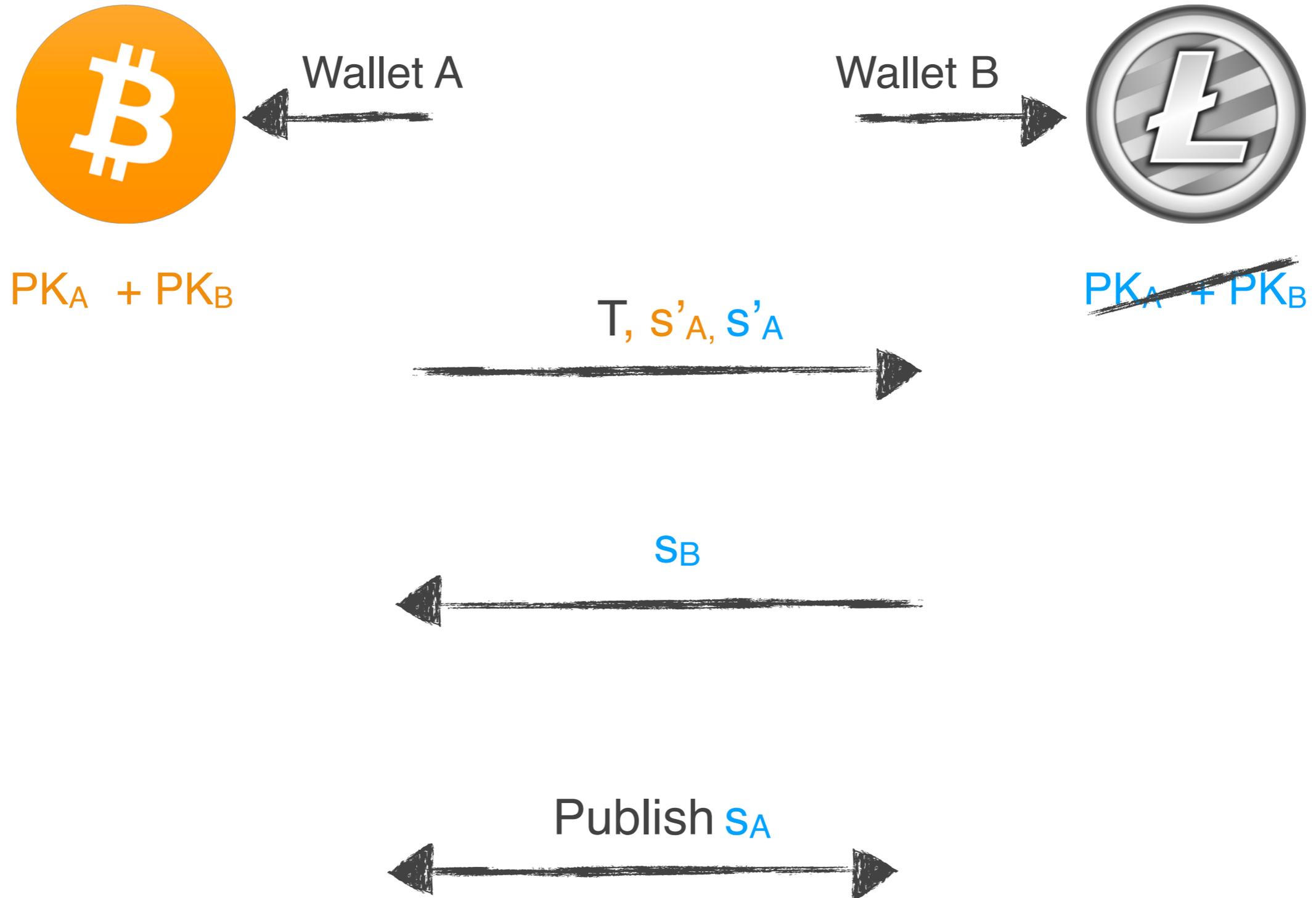
# SS Atomic Swap



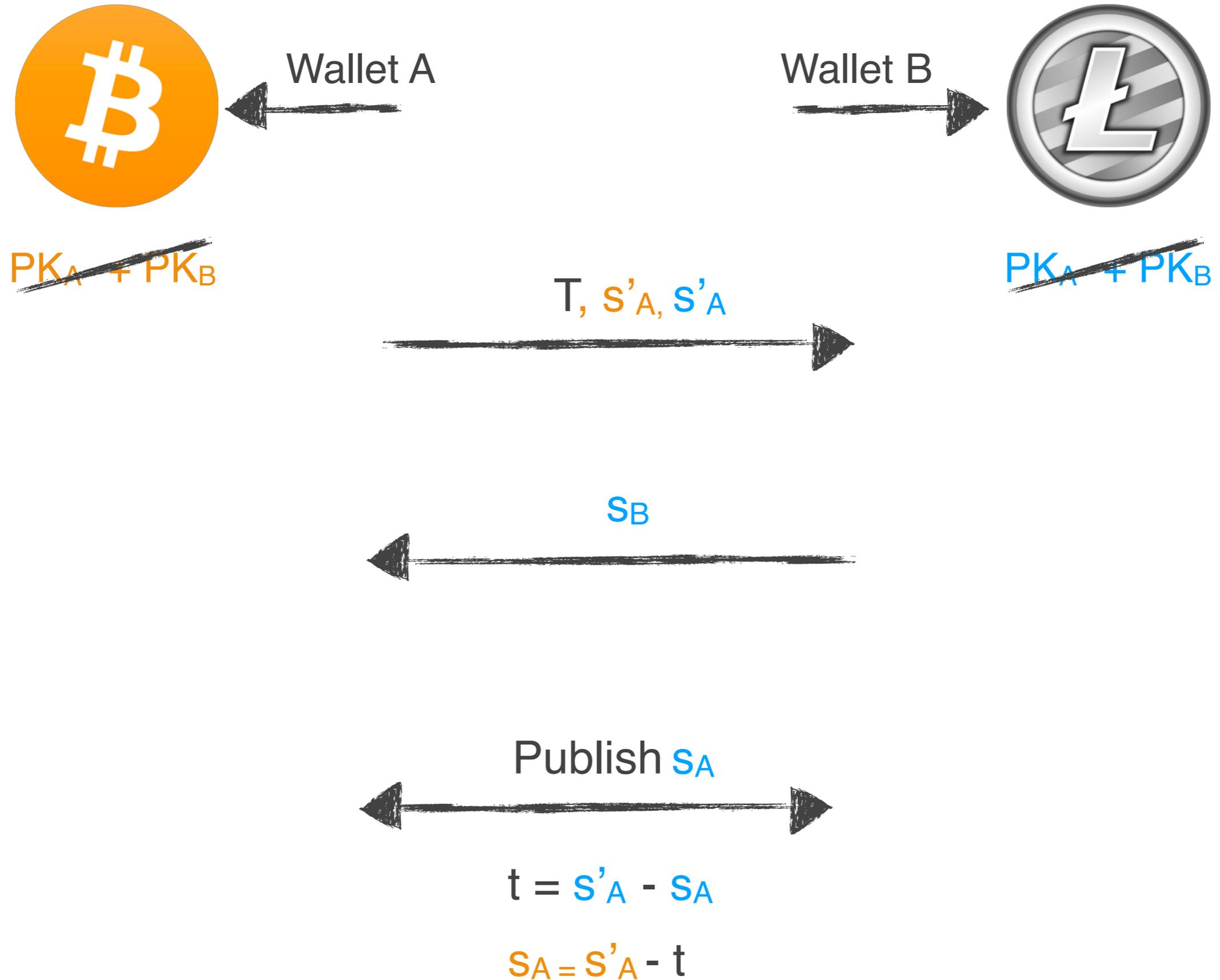
# SS Atomic Swap



# SS Atomic Swap



# SS Atomic Swap



# Scriptless Scripts

*Conclusion*

*Open Problems*

- Preserving scriptless scripts in multisig
- ECDSA support
- Locktimes and other extrospection
- Formalizing/understanding limits of scriptless scripts

# ECDSA SS - The Hard Questions

- Why we needed Schnorr in the first place?
- What are the challenges in using ECDSA for SS

# Why Schnorr ?

EC public parameters :  $q, G$

## ECDSA

- Choose random  $k$
- Compute  $R = k \cdot G$
- Compute  $r = r_x \bmod q$  where  $R = (r_x, r_y)$
- Compute  $s = k^{-1} \cdot (H(m) + r \cdot x) \bmod q$  where  $x$  is the private key
- Output  $(r, s)$

## Schnorr

- Choose random  $k$
- Compute  $R = k \cdot G$
- Compute  $s = k + H(R, P, m) \cdot x \bmod q$  where  $x$  is the private key,  $R = x \cdot G$
- Output  $(R, s)$

# Why Schnorr #2

## ECDSA:

- No security proof
- Malleable
- Not linear

## EC-Schnorr:

- Provably secure under ROMDL
- Provably non-malleable
- Linearity!

CIW19 Keynote



By Pieter Wuille  
March 20, 2019

# Why Schnorr #2

## ECDSA:

- No security proof
- Malleable
- Not linear

## EC-Schnorr:

- Provably secure under ROMDL
- Provably non-malleable
- Linearity!

CIW19 Keynote



By Pieter Wuille  
March 20, 2019

# ECDSA is a NO-GO?

# ECDSA: No security proof?

# ECDSA: No security proof?

**The Security of DSA and ECDSA**  
**Bypassing the Standard Elliptic Curve Certification**  
**Scheme**

Serge Vaudenay

Swiss Federal Institute of Technology (EPFL)  
Serge.Vaudenay@epfl.ch

- 2003, Generic Group Model

# ECDSA: No security proof?

## On the Provable Security of (EC)DSA Signatures

Manuel Fersch  
manuel.fersch@rub.de

Eike Kiltz  
eike.kiltz@rub.de

Bertram Poettering  
bertram.poettering@rub.de

Horst Görtz Institute for IT Security  
Ruhr University Bochum, Germany

- 2016, Bijective Random Oracle (BRO) model

# ECDSA: No security proof?

- ECDSA is one of the widely used signature schemes: TLS, PGP, S/MIME, multiple cryptocurrencies etc..
- ECDSA is widely standardized: IEEE P1363, ANSI X9.62, FIPS 186-4
- It was subject to massive cryptanalytic efforts with zero known attacks

## Implementations [\[ edit \]](#)

---

Below is a list of cryptographic libraries that provide support for ECDSA:

- Botan
- Bouncy Castle
- cryptlib
- Crypto++
- libgcrypt
- OpenSSL
- wolfCrypt
- mbed TLS

# ECDSA: Malleable ?

- Known malleability:
  - $(r, s)$  and  $(r, q - s)$  are both valid signatures on message  $m$

# ECDSA: Malleable ?

- Known malleability:
  - $(r,s)$  and  $(r, q - s)$  are both valid signatures on message  $m$
- Define ECDSA' as ECDSA that fix this malleability
  - ECDSA' is strong unforgeable

# ECDSA: Malleable ?

- Known malleability:
  - $(r,s)$  and  $(r, q - s)$  are both valid signatures on message  $m$
- Define ECDSA' as ECDSA that fix this malleability
  - ECDSA' is strong unforgeable
- BIP62 (WIP):

## Low S values in signatures

The value  $S$  in signatures must be between  $0x1$  and  $0x7FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF 5D576E73 57A4501D DFE92F46 681B20A0$  (inclusive). If  $S$  is too high, simply replace it by  $S' = 0xFFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C D0364141 - S$ .

# ECDSA: Malleable ?

- Known malleability:
  - $(r,s)$  and  $(r, q - s)$  are both valid signatures on message  $m$
- Define ECDSA' as ECDSA that fix this malleability
  - ECDSA' is strong unforgeable

- BIP62 (WIP):

## Low S values in signatures

The value  $S$  in signatures must be between  $0x1$  and  $0x7FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF 5D576E73 57A4501D DFE92F46 681B20A0$  (inclusive). If  $S$  is too high, simply replace it by  $S' = 0xFFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C D0364141 - S$ .

- Segwit

# Schnorr: Malleable ?

Q:



A:

scheme	public key	first component	second component	sign. size
[Sc91]	$-d \times G$	$H(Q, M)$	$k + d h$	$b + 2b$
EC-SDSA	$-d \times G$	$H(Q_x \parallel Q_y \parallel M)$	$k + d h$	$2b + 2b$
EC-SDSA-opt	$-d \times G$	$H(Q_x \parallel M)$	$k + d h$	$2b + 2b$
EC-FSDSA	$-d \times G$	$Q_x \parallel Q_y$	$k + d H(Q_x \parallel Q_y \parallel M)$	$4b + 2b$
EC-Schnorr	$d \times G$	$H(M \parallel Q_x)$	$k - d h$	$2b + 2b$
libsecp256k1	$d \times G$	$Q_x$	$k - d H(Q_x \parallel M)$	$2b + 2b$

# Linearity

- Valid point!
- Deal breaker?

# Linearity

- Valid point!
  - Deal breaker?
- 
- Given the non-linearity of ECDSA, Are ECDSA Scriptless Scripts
    - Possible ?
    - Possible but **Inefficient** ?
    - Possible but with compromise on **Security** ?

# Linearity: Observation

- **Threshold ECDSA** has struggled with the same problem.
- Not surprisingly
  - Existing works for **ECDSA-SS** are based on threshold ECDSA protocol

# Scaling Bitcoin 2018 #1

## Instantiating Scriptless 2P-ECDSA

Fungible 2-of-2 Multisigs for Today's Bitcoin

**Conner Fromknecht**

Head of Cryptographic Engineering, Lightning Labs



# Scaling Bitcoin 2018 #2

## Anonymous Multi-Hop Locks for Blockchain Scalability and Interoperability

Giulio Malavolta<sup>\*§</sup>, Pedro Moreno-Sanchez<sup>\*¶†</sup>, Clara Schneidewind<sup>†</sup>, Aniket Kate<sup>‡</sup>, Matteo Maffei<sup>†</sup>  
<sup>§</sup>Friedrich-Alexander-University Erlangen-Nürnberg, <sup>†</sup>TU Wien, <sup>‡</sup>Purdue University

# Scaling Bitcoin 2018 #2

## Anonymous Multi-Hop Locks for Blockchain Scalability and Interoperability

Giulio Malavolta\*<sup>§</sup>, Pedro Moreno-Sanchez\*<sup>¶†</sup>, Clara Schneidewind<sup>†</sup>, Aniket Kate<sup>‡</sup>, Matteo Maffei<sup>†</sup>  
<sup>§</sup>Friedrich-Alexander-University Erlangen-Nürnberg, <sup>†</sup>TU Wien, <sup>‡</sup>Purdue University

### Scriptless Scripts (SS-Schnorr)

- ▶ Technique originally proposed by A. Poelstra
- ▶ “Encode” payment condition within the Schnorr signatures
- ▶ Unfortunately, Schnorr is not used yet in many cryptocurrencies
- ▶ In our work:
  - formal description and security analysis
  - scriptless scripts based on ECDSA

# Previous Work

- Common to both is **2P-ECDSA**:

## Fast Secure Two-Party ECDSA Signing\*

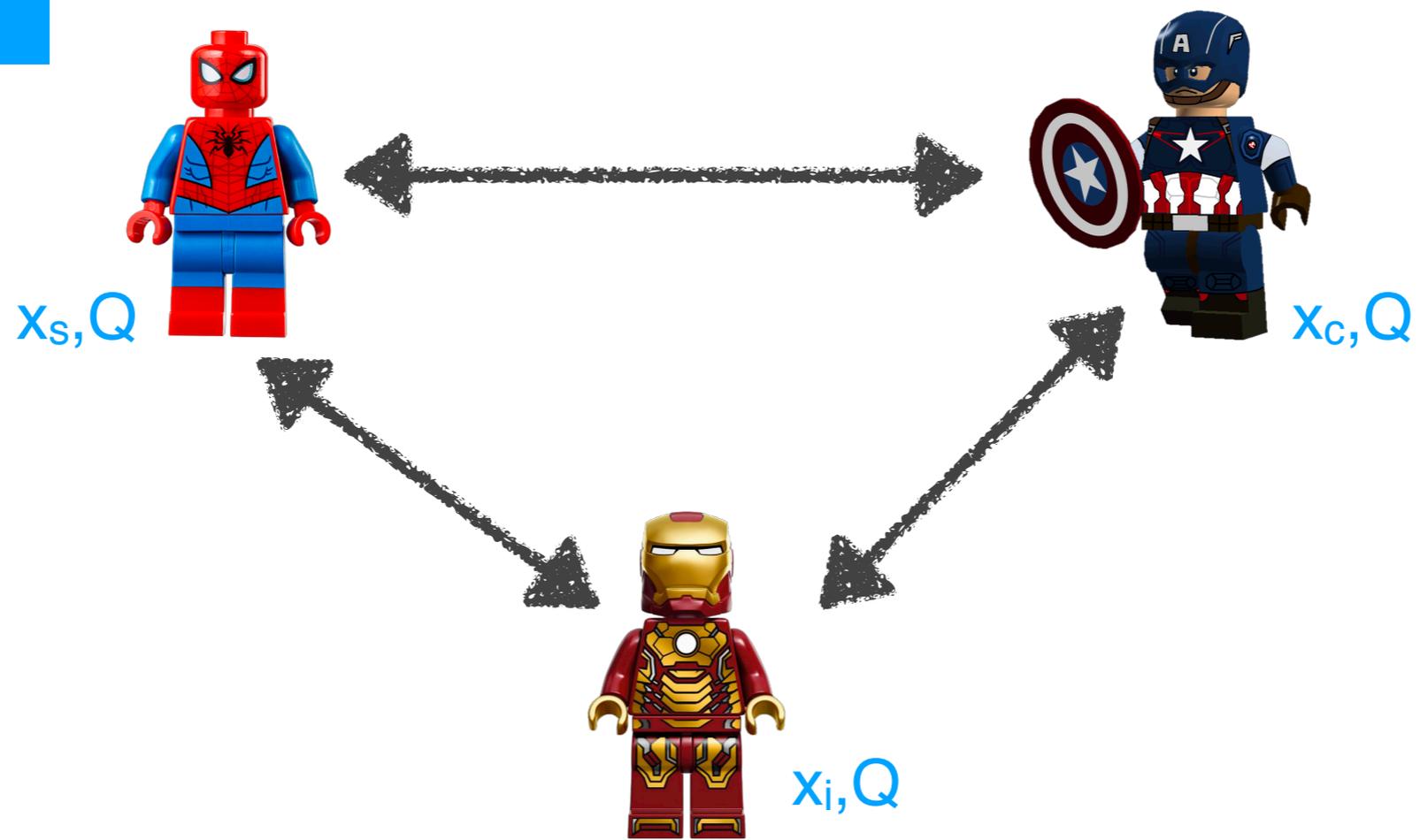
Yehuda Lindell\*\*

Dept. of Computer Science  
Bar-Ilan University, ISRAEL  
`lindell@biu.ac.il`

# Threshold Signatures

$(t,n)$ -threshold signature scheme distributes signing power to  $n$  parties such that any group of at least  $t$  parties can generate a signature

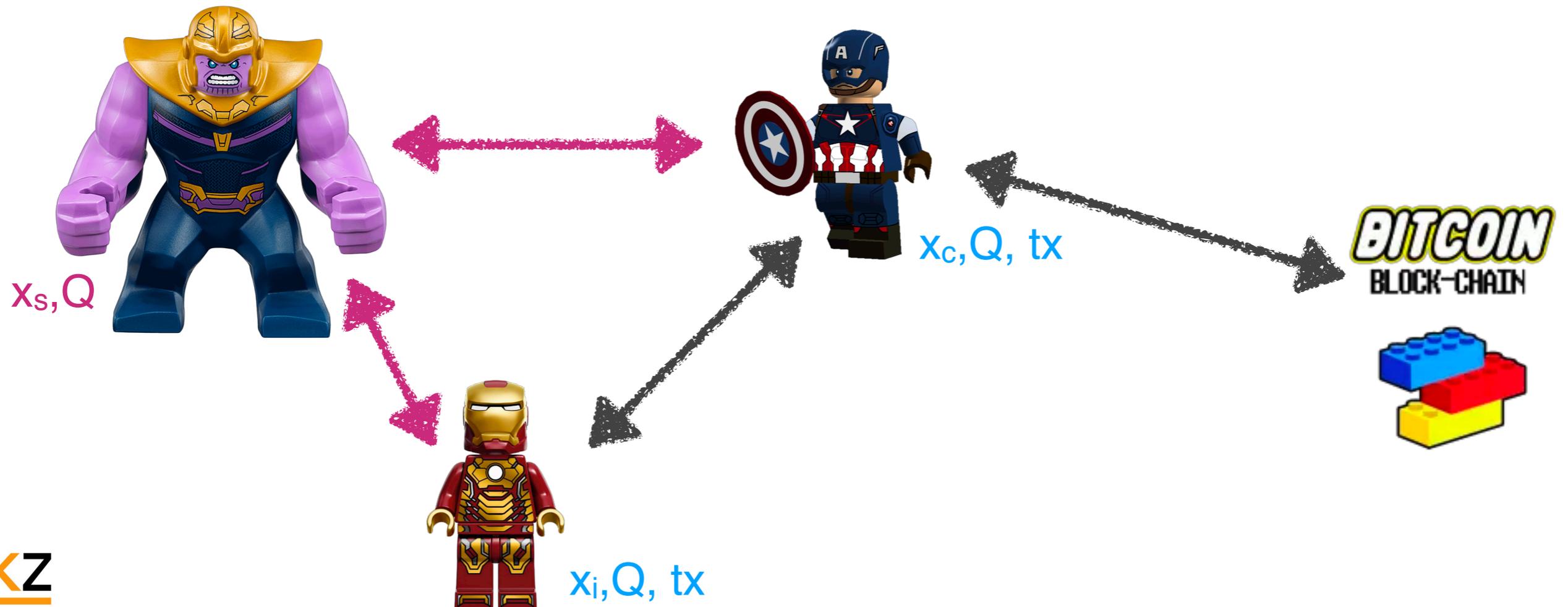
$(2,3)$  - Keygen



# Threshold Signatures

$(t,n)$ -threshold signature scheme distributes signing power to  $n$  parties such that any group of at least  $t$  parties can generate a signature

## (2,3) - Signing



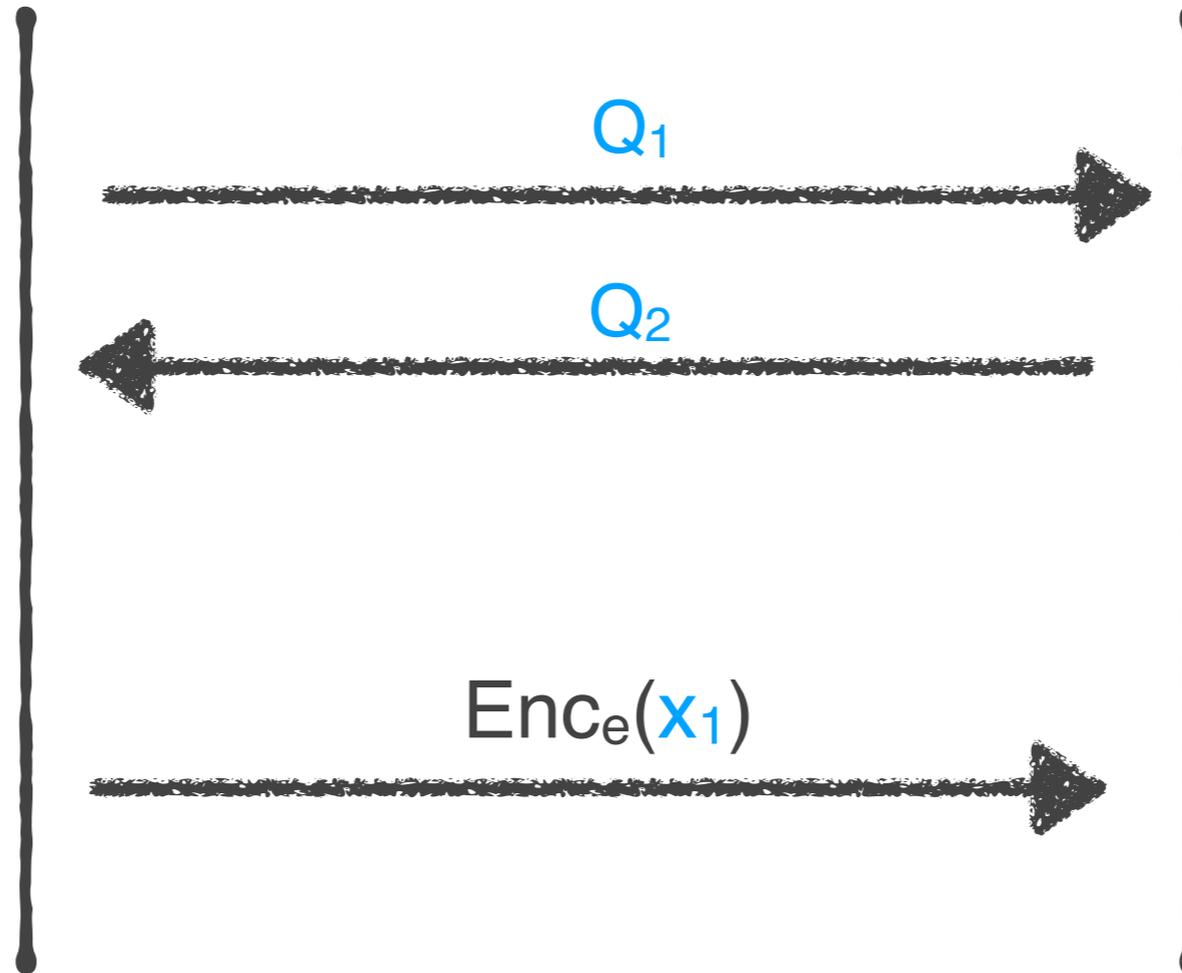
# 2P-Keygen [L17]



Party1

$$Q_1 = x_1 \cdot G$$

$$Q = x_1 \cdot Q_2$$



Party2

$$Q_2 = x_2 \cdot G$$

$$Q = x_2 \cdot Q_1$$

The protocol promises: (1) Privacy, (2) Correctness

# 2P-Signing [L17]

Signing message  $m$ :  $m' = \text{Hash}(m)$



Party1

$$R_1 = k_1 \cdot G$$

$$R = k_1 \cdot R$$

$$s = \text{Dec}_d(s^*) / k_1$$



Party2

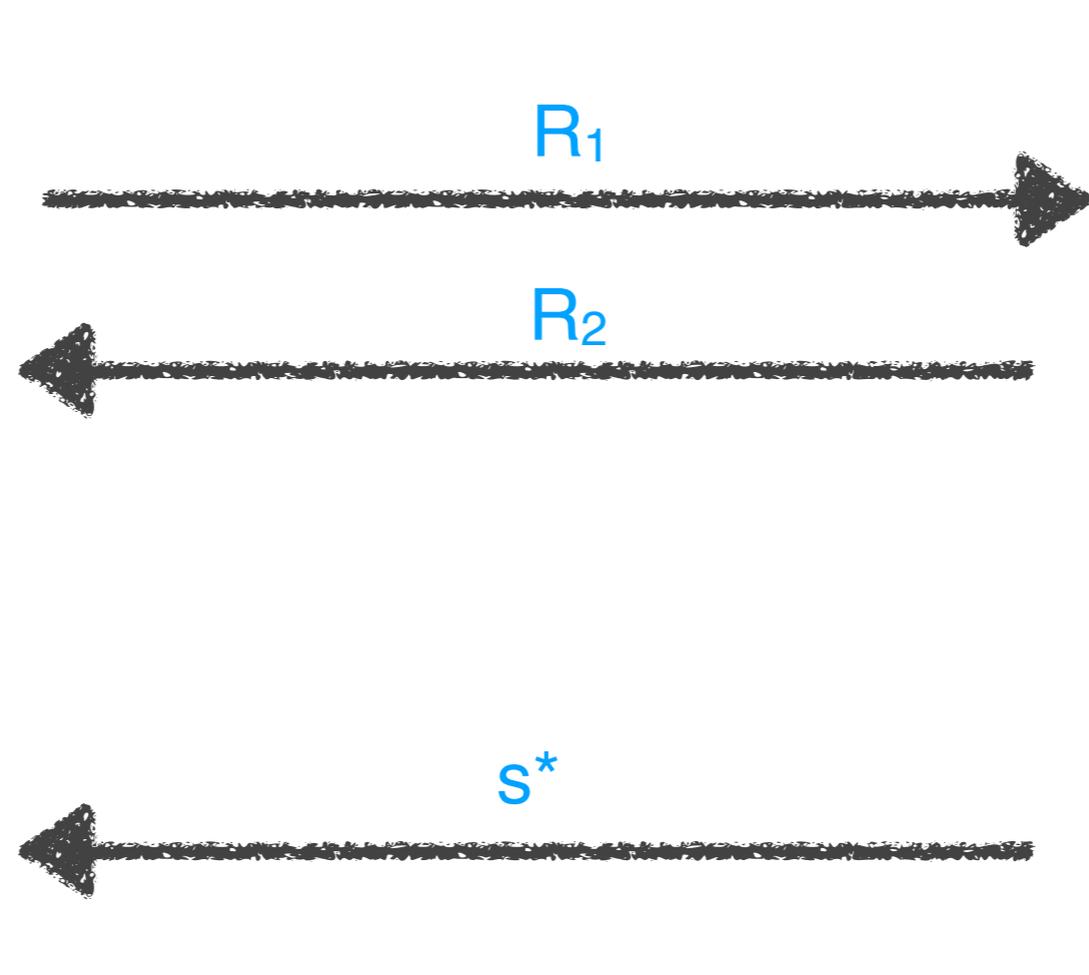
$$R_2 = k_2 \cdot G$$

$$R = k_2 \cdot R_1$$

$$s^* =$$

$$\text{Enc}_e(m' / k_2) \boxplus$$

$$\text{Enc}_e(x_1) \odot \text{Enc}_e(x_2 \cdot r / k_2)$$



Output:  $\sigma = (s, r)$ , s.t.  $\text{Verify}(\sigma, Q, m') = 1$

The protocol promises: Unforgeability



[KZen-networks/multi-party-ecdsa](https://github.com/KZen-networks/multi-party-ecdsa)

# 2P-ECDSA Lock [MMSKM18]

message  $m' = \text{Hash}(m)$ , Adaptor  $(t, T)$



Party1

$$R_1 = k_1 \cdot T$$

$$R = k_1 \cdot R$$

$$s' = \text{Dec}_d(s^*)/k_1$$



Party2

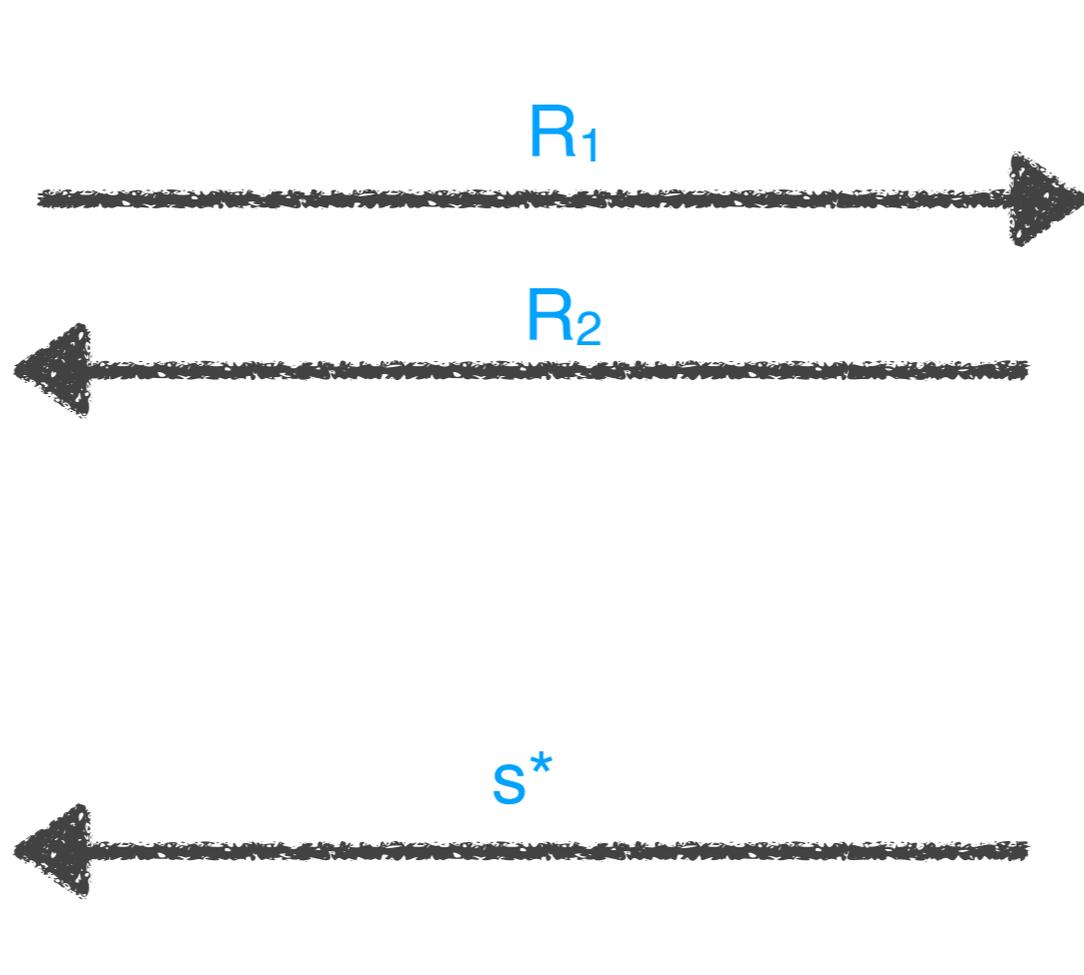
$$R_2 = k_2 \cdot T$$

$$R = k_2 \cdot R_1$$

$$s^* =$$

$$\text{Enc}_e(m' / k_2) \boxplus$$

$$\text{Enc}_e(x_1) \odot \text{Enc}_e(x_2 \cdot r / k_2)$$



Output:  $\sigma' = (s' = s \cdot t, r)$ , s.t.  $\text{Verify}(s' \cdot t^{-1}, r, Q, m') = 1$

# Possible Issues

- Given the non-linearity of ECDSA, Are ECDSA Scriptless Scripts
  - Possible ? **YES**, using Lindell 2P-ECDSA
  - Possible but **Inefficient** ? **MAYBE**
  - Possible but with compromise on **Security** ? **YES**

# Possible Issues #2

## Digital signing by utilizing multiple distinct signing keys, distributed between two parties

### Abstract

Described herein is a method and system for digital signing by utilizing Elliptic Curve Digital Signature Algorithm (ECDSA) with a group generator of an elliptic-curve group of order and an elliptic curve point  $Q$ . The method may be configured to receive a digital message and associated with a request from a third-party in order to sign the digital message. The system designed to sign such messages may comprise two parties denoted  $P1$  and  $P2$  configured to conduct a multiparty signing procedure by utilizing ECDSA. The digital signing procedure may follow preliminary steps configured to set the system with the necessary conditions for the multiparty signing procedure. Such preliminary steps may set the parties  $P1$ , and  $P2$ , in accordance with the configuration defined herein.

US20180359097A1

United States



Download PDF



Find Prior Art

**Inventor:** [Yehuda LINDELL](#)

**Current Assignee :** [Bar Ilan University](#)

# Bad News?

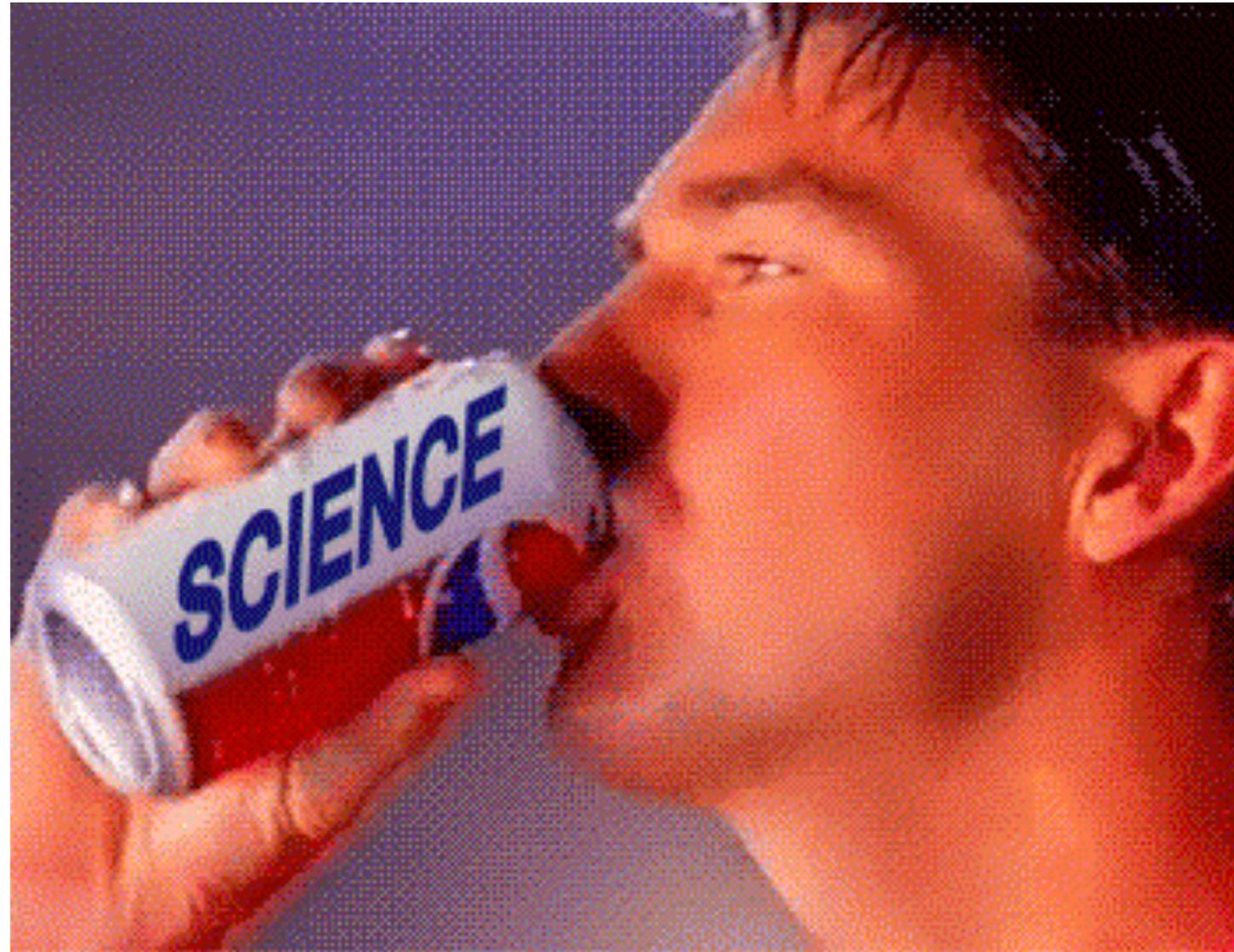
# Threshold ECDSA Papers Circa 2017-Today

	Params	Assumptions	Signing Rounds	Signing Time
[L17]	2/2	ECDSA, Paillier	4	milliseconds
[GG18]	t/n	ECDSA, Strong RSA	9	milliseconds
[LNR18]	t/n	ECDSA, DDH	8	milliseconds
[DKLS18]	2/n	ECDSA	2	milliseconds
[DKLS19]	t/n	ECDSA	$\text{Log}(t) + 6$	milliseconds
[CCLST19]	2/2	ECDSA, Class groups	4	milliseconds
[SA19]	n/n	ECDSA	1	
[DKOSS19]	t/n	ECDSA	1	sub millisecond

# Threshold ECDSA Papers Circa 2017-Today

	Params	Assumptions	Signing Rounds	Signing Time
[L17]	2/2	ECDSA, Paillier	4	milliseconds
[GG18]	t/n	ECDSA, Strong RSA	9	milliseconds
[LNR18]	t/n	ECDSA, DDH	8	milliseconds
[DKLS18]	2/n	ECDSA	2	milliseconds
[DKLS19]	t/n	ECDSA	$\text{Log}(t) + 6$	milliseconds
[CCLST19]	2/2	ECDSA, Class groups	4	milliseconds
[SA19]	n/n	ECDSA	1	
[DKOSS19]	t/n	ECDSA	1	sub millisecond

# Experiments



# Use Case #1: Scriptless Script MultiSig

- Access policy privacy
- Cost: one standard transaction
- Max number of parties

# Use Case #2: Threshold Wallet

- Distributed key generation (DKG)
- Distributed Signing
- Secret Share Recovery
- Deterministic Child Address Derivation
- Rotation



[KZen-networks/gotham-city](https://github.com/KZen-networks/gotham-city)



[unbound-tech/blockchain-crypto-mpc](https://github.com/unbound-tech/blockchain-crypto-mpc)

# Use Case #3: Coin Join Mixer

- Pubkey is equal to a sum of locally generated public keys:
  - $Pk = Pk_1 + Pk_2 + \dots + Pk_n$
  - Basically a constructed way for parties to reach off-chain consensus on output addresses like Chaumian coin-join
- No need for central coordinator

# Use Case #4: Atomic Swaps

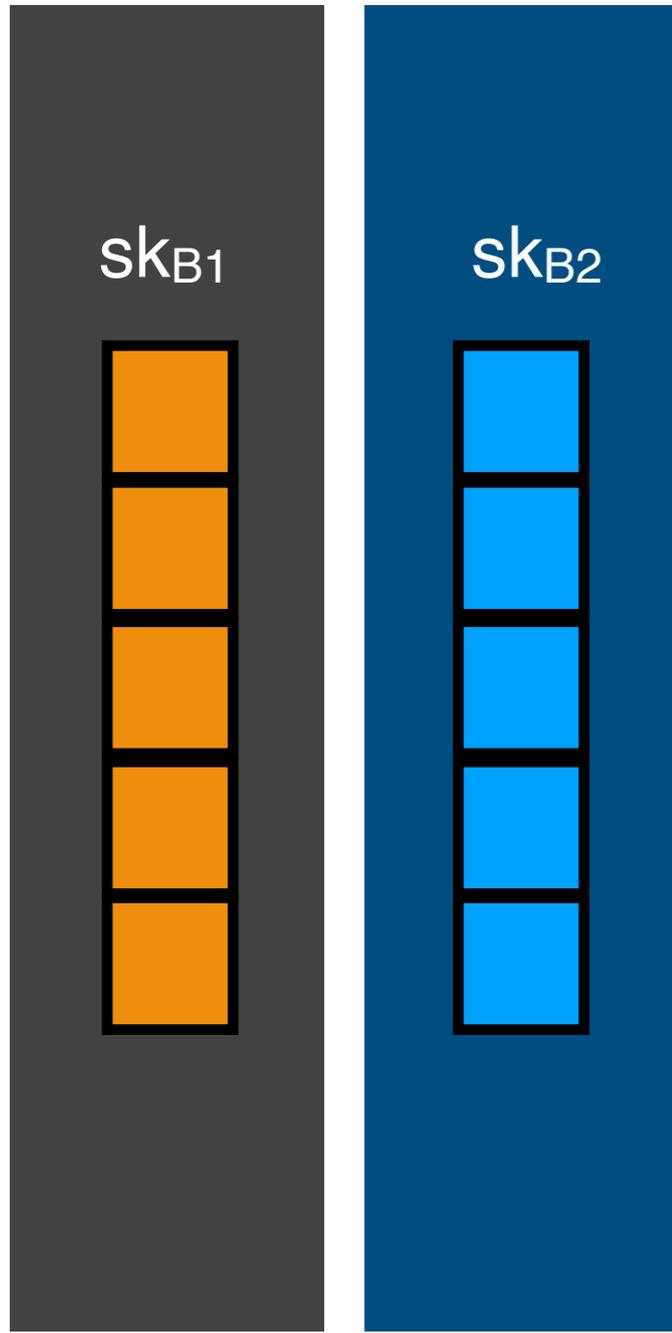
- Option 1: Use the scriptless script construction
  - Locking using Adaptor signatures.
- Option 2: Depends on access structure secret shares can be swapped using “gradual release of secrets”



[scriptless-scripts/atomic-swap](https://github.com/scriptless-scripts/atomic-swap)

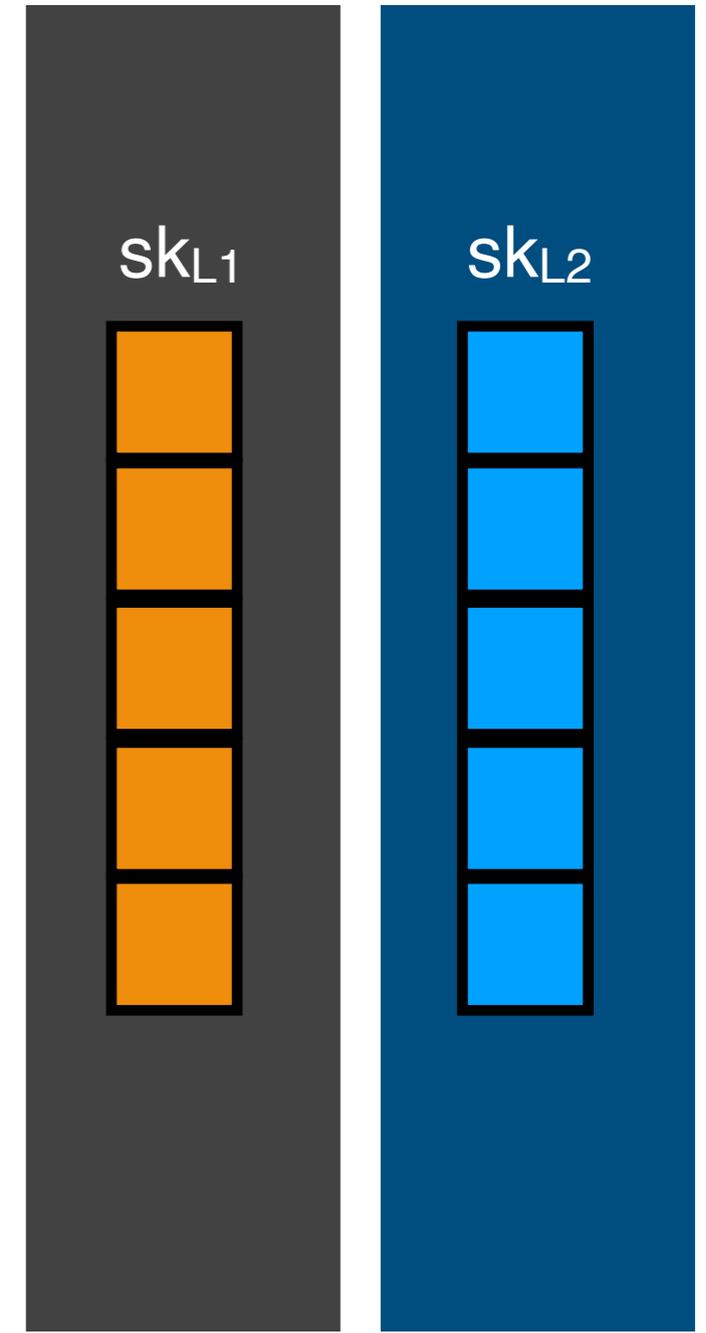


[KZen-networks/centipede/](https://github.com/KZen-networks/centipede/)



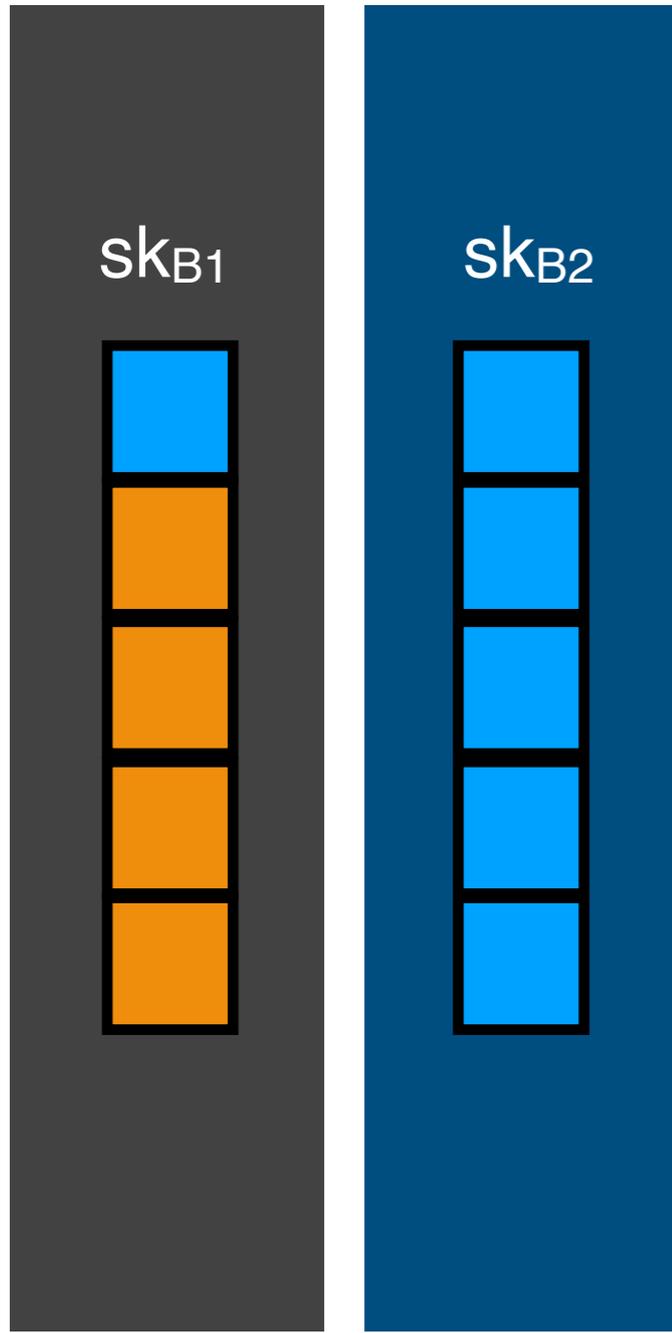
Wallet A

Wallet B



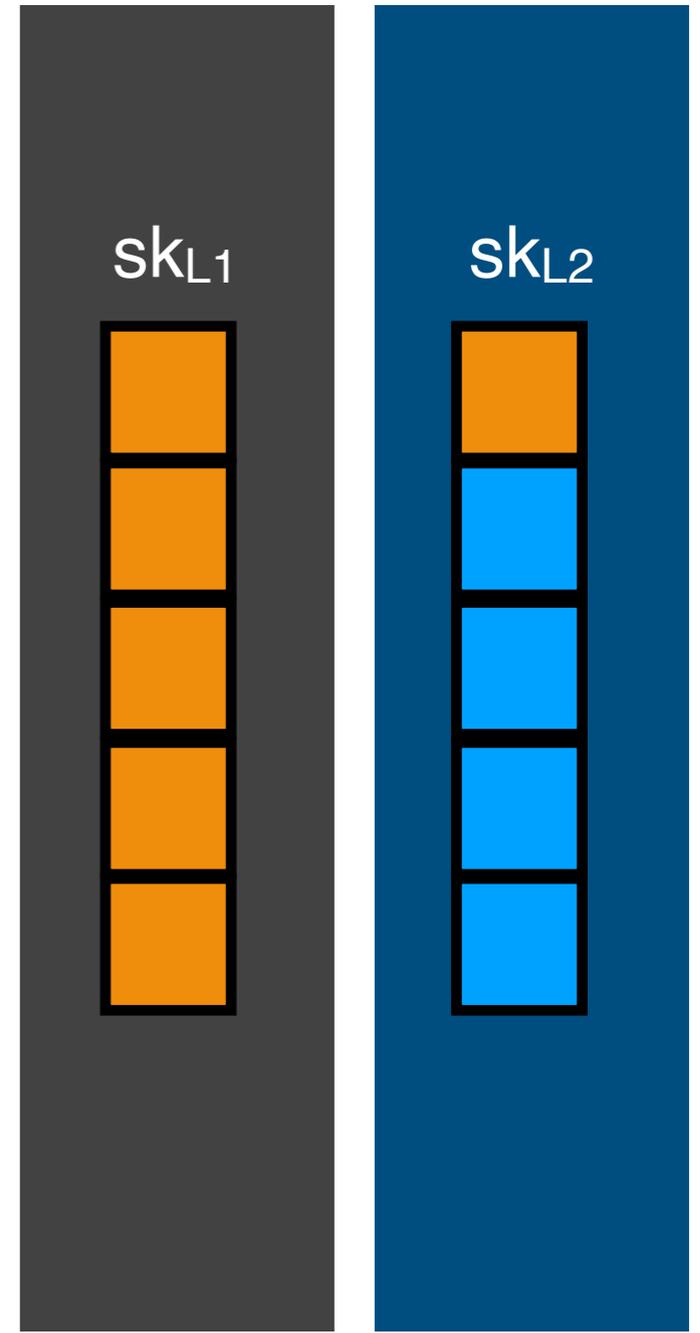
Wallet A

Wallet B



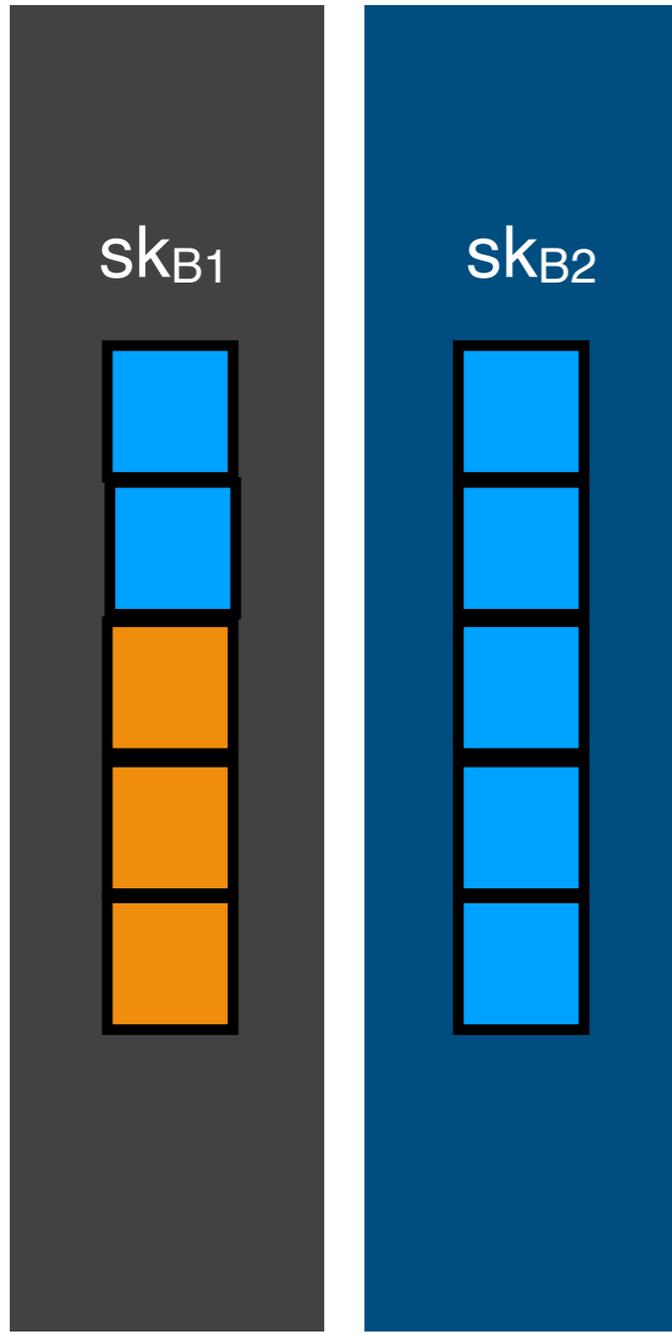
Wallet A

Wallet B



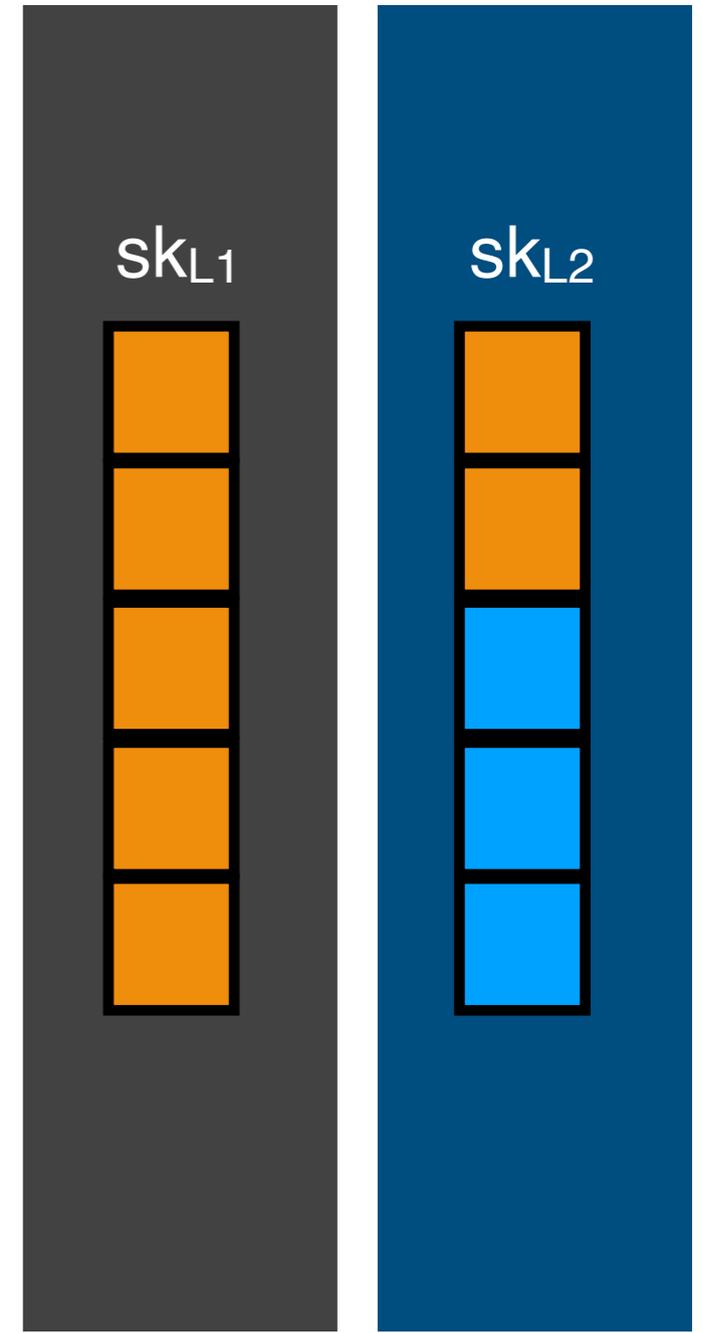
Wallet A

Wallet B



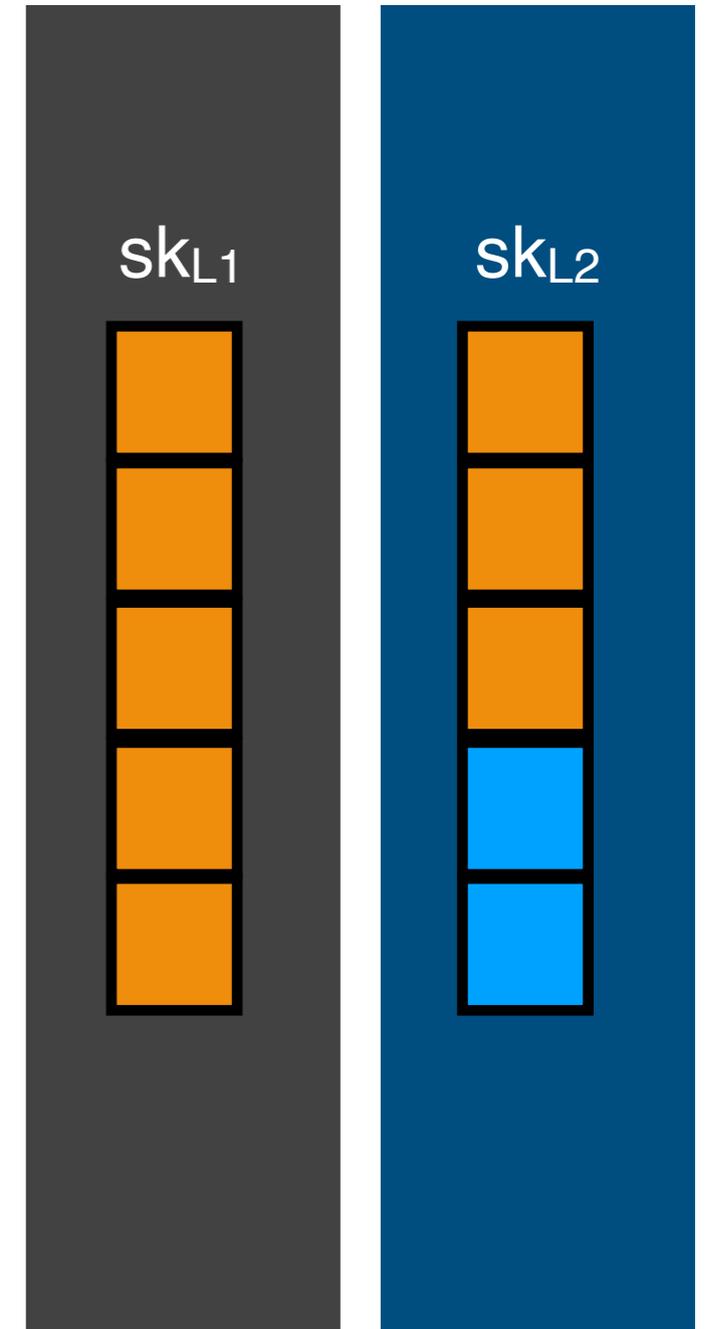
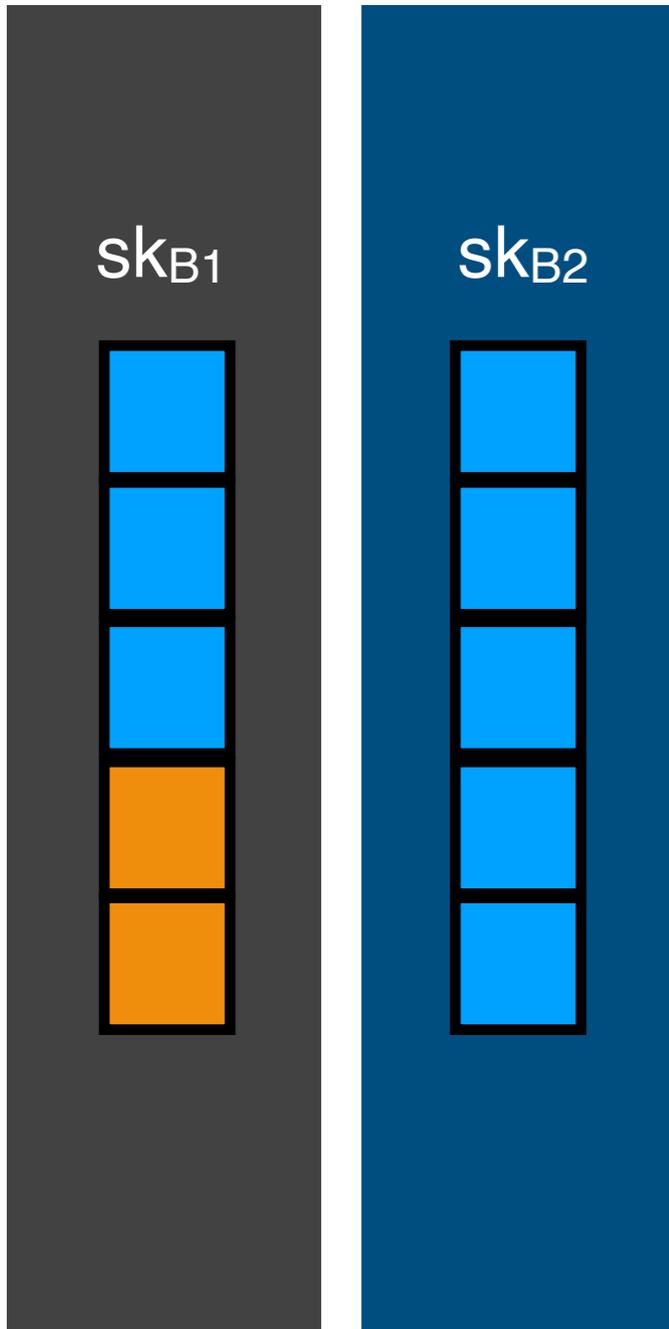
Wallet A

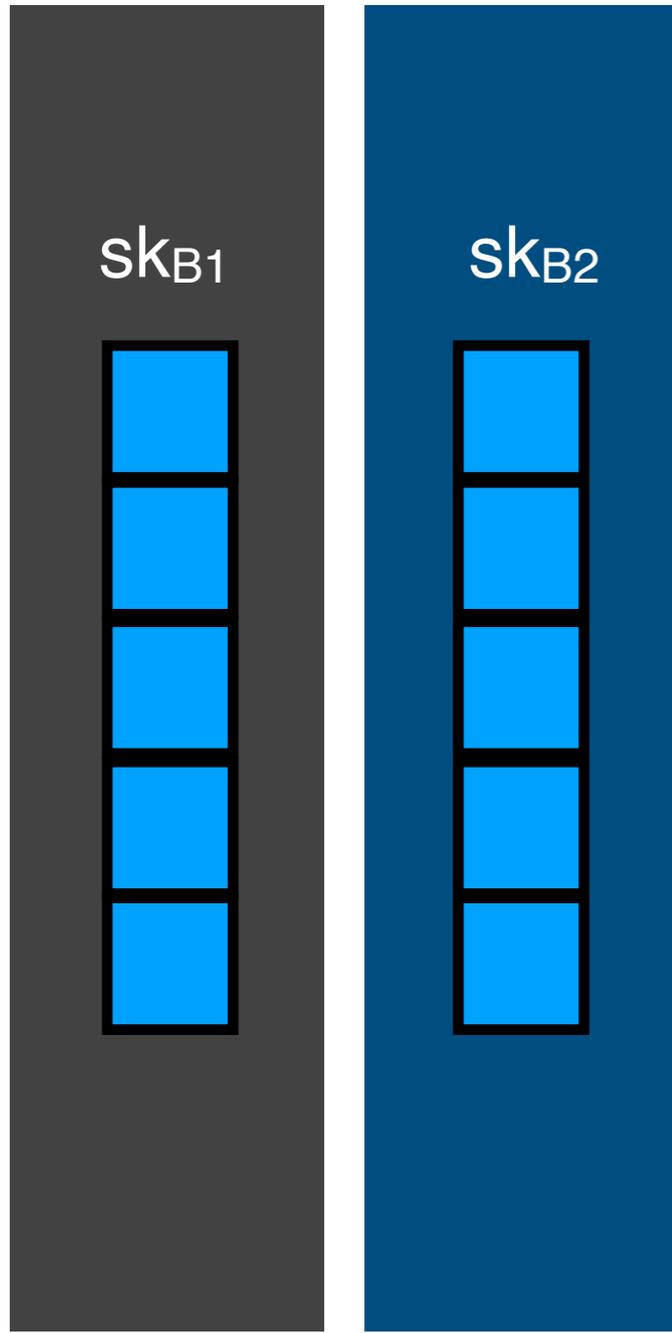
Wallet B



Wallet A

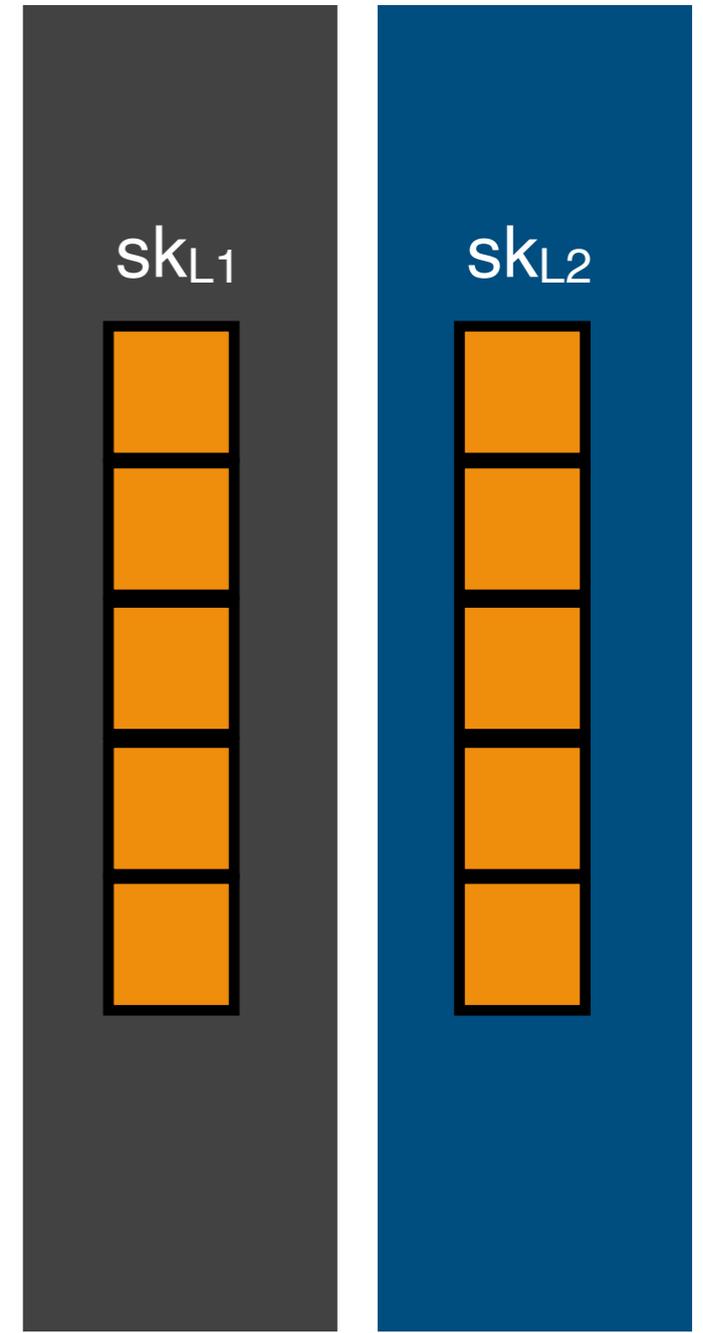
Wallet B





Wallet A

Wallet B



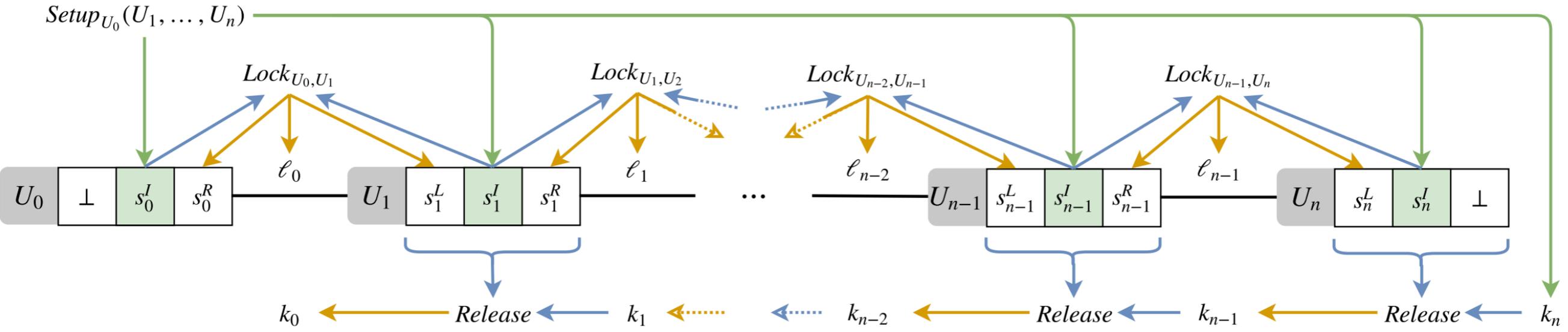
Wallet A

Wallet B

# Use Case #6: Payment Channel Network

## Anonymous Multi-Hop Locks for Blockchain Scalability and Interoperability

Giulio Malavolta<sup>\*§</sup>, Pedro Moreno-Sanchez<sup>\*¶†</sup>, Clara Schneidewind<sup>†</sup>, Aniket Kate<sup>‡</sup>, Matteo Maffei<sup>†</sup>  
<sup>§</sup>Friedrich-Alexander-University Erlangen-Nürnberg, <sup>†</sup>TU Wien, <sup>‡</sup> Purdue University



# Use Case #7: Zero Knowledge Contingent Payments

## **Efficient Zero-Knowledge Contingent Payments in Cryptocurrencies Without Scripts**

Wacław Banasik, Stefan Dziembowski, and Daniel Malinowski

University of Warsaw

# ZKCP

Prover / Seller

Knows  $x$



Broadcast TX<sub>2</sub>

Run threshold KeyGen



Sign TX<sub>2</sub>



ZK Prove  $x$  is encrypted by  $\sigma$



Verifier / Buyer



Generate TX<sub>1</sub>, TX<sub>2</sub>:

TX<sub>1,in</sub> : Buyer

TX<sub>1,out</sub> : threshold-Sig

TX<sub>2,in</sub> : TX<sub>1</sub>

TX<sub>2,out</sub> : Seller

Broadcast TX<sub>1</sub>

Extract  $x$

# R & D

Still long way to go!

# R & D

Still long way to go!

- Threshold cryptography - In standardisation process by NIST (\*)

# R & D

Still long way to go!

- Threshold cryptography - In standardisation process by NIST (\*)
- Network layer : authenticated secure p2p communication, Broadcast channel
  - Idea: use a blockchain/ consensus layer for the communication

# R & D

Still long way to go!

- Threshold cryptography - In standardisation process by NIST (\*)
- Network layer : authenticated secure p2p communication, Broadcast channel
  - Idea: use a blockchain/ consensus layer for the communication
- Improvements: Accountability , Batch signing and verification

# Summary

- Threshold ECDSA based Scriptless Scripts / systems
  - Are possible now
  - Are practical to use in real life
  - Hold strong security guarantees and the focus of active research in the cryptography community

# Summary

- Threshold ECDSA based Scriptless Scripts / systems
  - Are possible now
  - Are practical to use in real life
  - Hold strong security guarantees and the focus of active research in the cryptography community

## Questions?

Special thanks: Oded Leiba, Jonas Nick,  
Elichai Turkel, Pedro Moreno Sanchez



[ZenGo/research](https://zen.go/research)



[https://t.me/kzen\\_research](https://t.me/kzen_research)



<https://github.com/KZen-networks>